DESIGN OF REINFORCED CONCRETE ARCH RIB BRIDGE

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A THESIS

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Before proceeding with the design of the structure, it will be well to state clearly the factor that led to the selection of tho proposed site.

The drainage canal in Evanston, Illinois, which was constructed under the direction of the Chicago Sanitary District Commission for the purpose of diverting the sewage into the drainage canal, is spanned by a steel bridge a short distance from the lake. This bridge is used by an electric railway which runs north to Waukegan.

The Evanston Golf Links lie along the lake and extend west as far as the bridge, which looms up in all its ugliness to mar the otherwise picturesque landscape. It is proposed to design a bridge which will carry railroad traffic and yet not invoke criticism from the architect. This consideration entails a discussion of the relative merits of steel bridges and reinforced concrete structures.

Reinforced concrete was first considered merely a cheap substitute for stone, but its own merits are now recognized and it is used in a

manner according to its properties.

The fundamental principal of architectural design demands that the imitation of one material by the use of another shall not be made, and, therefore, in designing concrete bridges there should be no effort to imitate stone. The design should be treated truthfully and simply, keeping all lines in harmony with the material used.

The extent to which concrete and reinforced concrete are now being used in preference to stone or steel may be judged from the fact that, during the year 1908, there was at least twenty times as much cement manufactured and sold than in the corresponding period ten years previous. As methods of design and construction become generally understood and as workmen become more accustomed to handling concrete, there will be a still greater number of bridges built of this material. Long spans of three or four hundred feet or more will probably continue to be framed in metal, but there is reason to believe that all ordinary town and country bridges, and the major-

ity of railroad bridges, will be built as permanent structures.

Reinforced concrete is a good combination of materials. Concrete has a high compressive strength, but is weak in tension. Steel rods imbedded in concrete have a high tensile strength, but are weak in compression. The steel, therefore, strengthens the concrete, and the concrete stiffens the steel. The strength of one thus supplements the weakness of the other.

Some of the advantages of concrete bridges over steel bridges may be enumerated as follows: Cement hardens with age, and consequently the older the bridge, the stronger it becomes. Therefore, if it will successfully withstand its first test load, it will always be secure. This condition is reversed in steel structures, which deteriorate with age through the action of rust and the loosening of rivets and pins. As travel increases, concrete bridges become stronger to support it; neither is there any yearly expense for painting or maintenance.

Arches in general, which form is usually adopted for reinforced concrete bridges, present a more substantial and pleasing appearance than can be secured by any form of truss, even though an arched truss be considered, for in a truss the outline of the arch is not so evident as in a solid structure.

Again, concrete bridges have no noise or vibration, and require very little skilled labor. A concrete arch bridge so designed that tension cannot occur at any time, or under any condition of loading, is the most permanent bridge. If no tension occurs, cracks will not form to permit moisture to reach and corrode the reinforcing steel, and when the steel is permanently protected from the atmosphere and moisture, it should endure for centuries.

For all ordinary locations and length of span, there appears to be no good or sufficient reason for building unsightly frame structures when more permanent and artistic ones can be made at the same cost.

Design.

The method of analysis followed is that developed by Messrs. Turneaure and Maurer of the University of Wisconsin. The method is based on the elastic theory and is of general application to arches of variable moment of inertia and loaded in any manner. The analysis is mainly algebraic, although certain graphic aids may be used advantageously.

To determine the length of the final divisions of the arch ring, a base line is drawn equal in length to the semi-axis of the arch rib. This line is divided into a number of parts equal to the number of preliminary divisions in one half of arch. At the middle of each division ordinates are erected equal to the moment of inertia of the corresponding division. A smooth curve is drawn through the extremities of these ordinates. Commencing at one end, a line is drawn at any slope, and from the point where it cuts the curve another line is drawn at the same slope. This process is continued across the diagram. If the sloped

lines do not divide the base into ten parts (the number of preliminary divisions), another slope must be chosen and the process repeated until the base line is divided into ten parts.

The analysis of an arch consists in the determination, in position and magnitude, of the resultant of all the forces acting at the section; and the determination of the stresses resulting therefrom. The resultant force is usually resolved into a normal thrust acting at the gravity axis of the section, a transverse shear, and bending moment. The thrust is the component of the resultant parallel to the arch axis at the given point, and the shear is the component at right angles to such axis.

The method of procedure will be to determine first, the thrust, shear, and bending moment at the crown. These being known, the values of similar quantities for any other section can readily be determined. A length of arch of one unit will be considered.

Notation.

Let -- H_o thrust at the crown

V_o shear at the crown

M_o bending moment at the crown

assumed as positive when causing compres-

sion in the upper fiber.

N, V, and M = thrust, shear, and moment at any other section.

R = resultant pressure at any section = resultant of N & V.

δs = length of a division of the arch ring measured along the arch axis.

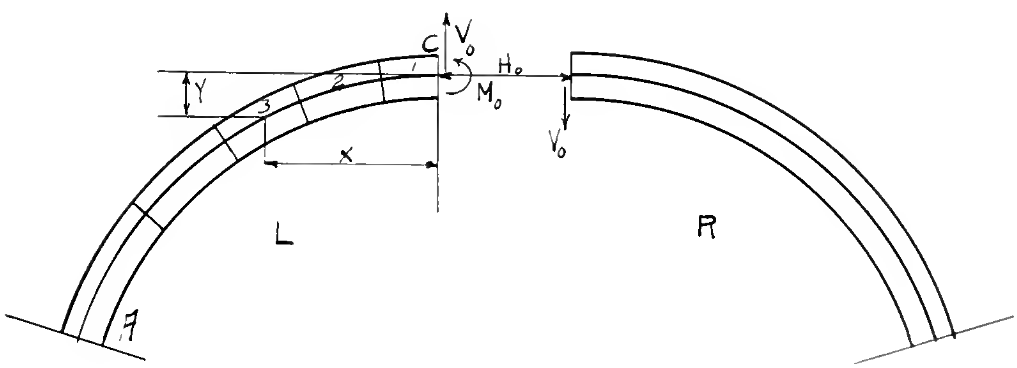
n = number of divisions in one-half of the arch.

I = moment of inertia of any section = $I_{concrete} + nI_{steel}$.

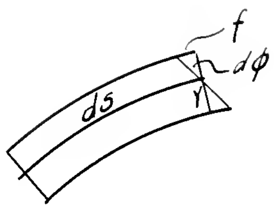
P = any load on the arch.

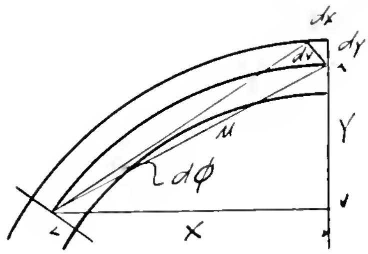
x, y = co-ordinates of any point on the arch axis referred to the crown as origin, and all to be considered as positive in sign.

m = bending moment at any point in the cantilever due to external loads.



Under the forces acting, the point C (see sketch) will deflect and the tangent to the axis at this point will change direction (the abutment at A being fixed). Let Δy, Δx and $\Delta\phi$ be, respectively, the vertical and horizontal components of this motion and the change in the angle of the tangent.





$$E = \frac{f}{\frac{Y d\phi}{ds}} = \frac{f\,ds}{Y d\phi}$$

$$f = \frac{E Y d\phi}{ds} = \frac{MY}{I}$$

$$(1) \quad d\phi = \frac{ds}{EI} M$$

$$(2) \quad \Delta\phi = \frac{ds}{EI} \Sigma M$$

$$dv = v\,d\phi \qquad \frac{dv}{dx} = \frac{v}{Y}$$

$$dv = \frac{v\,dx}{Y} = v\,d\phi$$

$$d\phi = \frac{dx}{Y} \quad \text{and} \quad dx = Y d\phi$$

Substitute in Eq (1)

$$dx = Y M \frac{ds}{EI}$$

Substitute in Eq (2)

$$\Delta X = \frac{ds}{EI} \Sigma MY$$

IN LIKE MANNER

$$\Delta Y = \frac{ds}{EI} \Sigma MX$$

$$\therefore \quad \Delta Y = \frac{ds}{EI} \Sigma MX \;, \quad \Delta X = \frac{ds}{EI} \Sigma MY \;, \quad \Delta\phi = \frac{ds}{EI} \Sigma M \qquad (a)$$

In liko manner, referring to the right cantilever, let $\Delta y'$, $\Delta x'$, and $\Delta\phi'$ represent the components of the movement of C and the change of angle of the tangent. These may be expressed in terms similar to equation (a)

$$\Delta Y = \Delta Y', \quad \Delta X = -\Delta X' \quad \Delta\phi = -\Delta\phi' \qquad (b)$$

Since $\frac{ds}{I}$ and E are constants, the quantity $\frac{ds}{EI}$ may be placed outside tho summation sign.

Using the subscript L to donote left side and R to donote right side, we then derive the relations

$$\left.\begin{aligned} \Sigma M_L X &= \Sigma M_R X \\ \Sigma M_L Y &= \Sigma M_R Y \\ \Sigma M_L &= -\Sigma M_R \end{aligned}\right\} \qquad (c)$$

The moment M may in general be expressed in terms of known and unknown quantities thus:

$$M_L = m_L + M_o + H_o Y + V_o X \quad \text{For The Left Side}$$

$$M_R = m_R + M_o + H_o Y - V_o X \quad \text{For The Right Side}$$

Substituting in (c) and combining terms and noting that ΣM_o for one half is oqual to

nM_o we have

$$\Sigma m_L x - \Sigma m_R x + 2V_o \Sigma x^2 = 0 \qquad (d)$$
$$\Sigma m_L y + \Sigma m_R y + 2M_o \Sigma y + 2H_o \Sigma y^2 = 0 \quad (e)$$
$$\Sigma m_L + \Sigma m_R + 2nM_o + 2H_o \Sigma y = 0 \quad (f)$$

From equation (d) is obtained equation (2) and from equation (e) and (f) are obtained equations (1) and (3) noting that $\Sigma m_L + \Sigma m_R = \Sigma m$ and $\Sigma m_L y + \Sigma m_R y = \Sigma my$

$$H_o = \frac{n\Sigma my - \Sigma m \Sigma y}{2\left[(\Sigma y)^2 - n\Sigma y^2\right]} \qquad (1)$$

$$V_o = \frac{\Sigma (m_R - m_L)x}{2\Sigma x^2} \qquad (2)$$

$$M_o = -\frac{\Sigma m + 2H_o \Sigma y}{2n} \qquad (3)$$

The values of H_o, V_o, and M_o having been found, the total bending moment at any section is

$$M = m + M_o + H_o y \pm V_o x$$

The plus sign to be used for the left half and the minus sign for the right half of the arch.

From the values of thrust, moment, and eccentric distance, as given in the tables, the stresses in the concrete and steel can be found at any section of the arch, as explained in Chapter 3 of Turneaure and Maurer: Principles of Reinforced Concrete Construction.

Design of Abutments.

The abutments will be of concrete bonded directly to the arch rib so that the whole structure may act as an elastic monolithic structure. The reinforcement will consist of vertical rods arranged as shown on the detail drawing, together with part of the steel of the rib which extends 10 to 15 feet into the abutment and is bent up at the extremities. The dimensions of the abutment are determined by Trautwine's Rule (See Trautwine: Abutments). The depth below the springing line is 18 feet where the width becomes $31\frac{1}{4}$ feet. Using the formula $S = 6Ve \div d^2$, we get $(6 \times 4.65 \times 367000) \div (31\frac{1}{4})^2 = 10500$ lbs. per sq.ft. due to overturning pressure on the toe of the base. The direct pressure $V \div b = 367000 \div 31.25 = 11740$ lbs. per sq.ft. Adding we get the pressure at the toe 22,240 lbs. per sq.ft., and at the heel, subtracting, we get 1240 lbs. per sq.ft., values well within safe limits.

In order to determine the overturning moment and the eccentricity the resultant of

all the forces (namely, the weight of the abutment, the pressure of the earth, and the thrust of the arch rib), was obtained graphically.

For details of construction, see sheet No. 3.

Design of Spandrel Walls.

The spandrel walls are of 1:2:4 concrete, reinforced vertically with $\frac{3}{4}$ inch round rods, spaced 24" on centers. The maximum shear is over the abutments and is 26,600#. This is found by substituting in the formula $\frac{wh^2}{6}$, where w weight of cu.ft. of earth and h depth of fill over abutment. This will require a wall 4 feet thick at bottom of fill. This thickness will be tapered to 2 feet, as in section (See plate 3). The tensile stresses due to lateral pressure of earth is taken care of by the diaphragms which tie the walls together. Architectural details designed on drawings.

Design of Diaphragms.

For convenience in computation of the earth pressures, the span was divided up into sections

For computing the amount of horizontal steel in the spandrel walls to resist the earth pressure, consider a unit layer of the wall as a horizontal, continuous beam whose span is the distance between diaphragms. The horizontal pressure due to crushed rock filling, weighing 100 lbs. per cu.ft. at a depth of 20 ft., is 20 x 100 x .3 = 600 lbs. per square ft. This acts laterally against the beam as a uniform load, causing a bending moment of 635000 lb.ins. The thickness of the spandrel wall at this depth is 40 ins. and with the steel placed 4 ins. from the outside, "d" becomes 36 ins. The lever arm of the steel for rectangular beams is seven eighths of "d". Using 15000 lbs. per sq.in. as the working stress for steel, the necessary area is 1.6. sq.ins. This can be supplied by spacing one inch rods 7 ins. apart. By similar calculations at different depths, the following spacing was determined:

To depth of 6 ft. space rods 18 ins. apart
From 6 to 14 ft. 4 ins. space 10 ins. apart
from 14 ft.4 ins. to bottom space 7 ins. apart.

The concrete stress developed in the spandrel walls is less than three hundred pounds per sq.in., giving a large margin of safety.

and the depth of the top of arch rib below top of fill measured at center of each section. For exact location of diaphragms, see place 3, and for values of depth, see plate 1.

As an example of the design of diaphragms, we will compute the size and amount of steel for diaphragm over abutment. The value of the depth is 36.5 feet. The depth is taken at the center of a 12.5 foot section. Therefore, to get the total pressure to be resisted by the steel in tho diaphragm is found to be $\frac{wh^2}{6}$ 12.5 or 333,000#. To get the steel neccssary, we divide the thrust by the allowed tensile stress. This gives the required number of square inches. Using $\frac{5}{8}$" round rods, we find that 74 will be required.

In order to reduce the bending moment in the diaphragms due to their own weight, a support of 1:2:4 concrete reinforced with $\frac{3}{4}$" round rods is placed under the center. This reduces the bending moment by such an amount that only one $\frac{5}{8}$" round per sq.ft. will be needed to take care of tensile stresses due to bending moment.

The exact sizes of diaphragms are dependent on amount of steel required and spacing of same. And so will be left until details are drawn up, 1:2:4 concrete used.

Centering for Arch Rib.

Use of Centering.

During construction, the concrete arch rib must have adequate support, and this support will consist of a timber braced platform following the curved outline of the intrados. Owing to the time required for the concrete to set, the centering must be wide enough to support the whole structure, although the arch rib will be constructed in longitudinal sections from springing to springing, and only wide enough to enable the workmen to complete one section in each day's work.

Kind of Centering Used.

The arch rib will be supported during erection by a modified bowstring truss of the Warren type, with the upper chord curved like the intrados, and the lower chord horizontal.

The lower chord will lie on a line connecting the springing points of the arch. The material used will be Southern yellow pine of good quality. The joints will be made of straps of iron $2\frac{1}{2}$ inches wide, $\frac{1}{2}$ inch thick, and long enough to hold the required number of bolts, at four inch spacing.

Since the bridge is 32 feet wide, nine such trusses will be required when spaced four feet apart. Instead of allowing the lower chord to resist the heavy tensile stresses brought upon it by the web members, it will be supported beneath by a series of piers, as shown on drawing No. 4, thus relieving the lower chord of all tensile stress and transmitting the loads from the vertical struts directly to the piers below.

Loads upon the Centering.

Each truss will consist of 16 equal panels of 8 ft. 6 ins. each. Allowing 150 lbs. per cu.ft. for concrete and 40 lbs. per sq.ft. for men and tools of construction, the load coming upon each panel point, due to an area of 32

square feet, was computed and the values marked upon the drawing, as shown on sheet No. 4. The verticals will be designed to carry those respective loads and the diagonals will be designed as bracing to stiffen the trusses against stresses caused by wind and eccentric loading.

The longest upright, at the center, will be 28 ft., but this and the four uprights each side of it will be braced midway, and so the effective length of those verticals will be about 14 ft. By referring to a safe loads table in the Carnegie Handbook, it is found that a 6" x 6" post will carry a little over 8 tons, while the actual load is a little less than 8 tons. The sizes required to carry the other loads at points nearer the abutments differ so slightly from the above that economy will be gained by using throughout 6" x 6" yellow pine struts for the verticals.

The diagonals should be made of the same width as the verticals, so that the joints can be made quickly by means of iron straps; the other dimension may be reduced to four inches.

Therefore, use 6" x 4" yellow pine for all diagonals.

The lower chord will be made the same width as the struts above, but the depth will be increased to 8 inches. Each set of three verticals will be supported by a pier below by means of a triangular framework, or bent, as shown on sheet No. 4. The vertical timbers will be 8" x 8" and the diagonal timbers 6" x 6".

Each pier carries the loads coming from three struts from each of three trusses, as will be seen by referring to the drawing. This amounts to a total load of 9 x 7.2 tons, or 65 tons. By the Chicago Building Law, 2 tons per sq.ft. is the permissible pressure of a foundation upon clay. This requires an area of 33 sq.ft. and a pier having a base of 6 feet each way will amply support the load. As the piers are short compared to the width of base, a bearing pressure of 600 lbs. per square inch may be used for concrete. An area of 216 square inches will be required. To fulfill this re-

quirement, make the tops of the concrete piers 15 inches square.

Pier Beams.

As there are three piers to support nine struts, a beam must be placed across the tops of the piers and will sustain from the bents two concentrated loads of 22 tons each spaced four feet apart. The maximum span will be 20 feet. The bending moment = 4 x 12 x 44000 = 2,110,000 lb. ins. Safe stress for steel is 16000 lbs. per sq. in.

Section modulus = moment ÷ stress = 132. This modulus can be obtained by using a 20 inch steel "I" beam, 80 lbs. per foot. Section modulus 147.

Lagging.

The lagging, or platform, upon which the concrete is deposited will be made of tongued and grooved yellow pine flooring 6 inches wide and 2 inches thick. The thickness is determined by use of the flexure formula, using 2000 lbs. per sq.in. as the safe fiber stress for pine.